Isomorphic graphs with Myanmar Alphabet

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Abstract

In this paper, firstly, the fundamental concepts of graphs are studied. Next, the isomorphism of graphs and graphs of Myanmar alphabet are expressed. Finally, the isomorphic Myanmar alphabet graphs are discussed with some examples.

Keywords : graph, degree sequence, Myanmar alphabet, isomorphic graph, isomorphism.

1. Introduction

Graph theory is a branch of mathematics that has wide practical application. A graph is formed by points and lines connecting the points. A graph can exist in different forms having the same number of points, lines, and also the same line connectivity. In this paper, I present Myanmar Alphabets are presented by using isomorphic graphs.

2. Some Basic Definitions and Notations

A **graph** G is an ordered triple (V(G), E(G), ψ_G) consisting of a nonempty set V(G) of vertices (points), a set E(G), disjoint from V(G), of edges (lines), and an incidence function $\psi(G)$ that associates which each edge of G an unordered pair of (not necessarily distinct) vertices of G. The vertex set of G is denoted by $V(G) = \{v_1, v_2, \ldots, v_n\}$, while the edge set is denoted by $E(G) = \{e_1, e_2, \ldots, e_m\}$. The cardinality of the vertex set of a graph G is called the **order** of G and is commonly denoted by n(G), or more simply by n, sometimes denoted by ν. The cardinality of its edge set is the **size** of G and is denoted by m(G) or m, sometimes denoted by ε.

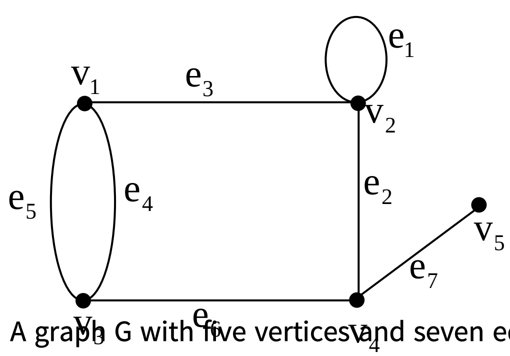

Figure (1) A graph G with five vertices and seven edges

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In Figure 1, $V(G)=\{v_1,v_2,v_3,v_4,v_5\}$, $E(G)=\{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$,

$n(G)=5$, $m(G)=7$.

The **edge** $e=\{u,v\}$ is said to join the vertices u and v. If $e=\{u,v\}$ is an edge of a graph G, then u and v are **adjacent vertices**, while u and e are **incident**, as are v and e. Furthermore, if e_1 and e_2 are distinct edges of G incident with a common vertex, then e_1 and e_2 are **adjacent edges**. An edge with identical ends is called a **loop**. An edge with distinct ends a **link**. **Parallel edges** or **multiple edges** are edges that have the same pair of endpoints

The **degree** $d_G(v)$ of a vertex v in G is the number of edges of G incident with v, each loop counting as two edges. We denote by $\delta(G)$ and $\Delta(G)$ the minimum and maximum degrees, respectively, of vertices of G.

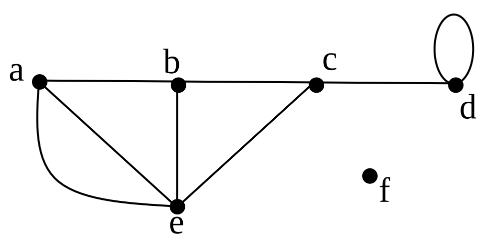


Figure (2) A graph G

In Figure 2,

d(a) = 3, d(b) = 3, d(c) = 3, d(d) = 3,

d(e) = 4, d(f) = 0, $\delta(G)=0$, $\Delta(G)=4$.

If G has vertices $v_1,v_2,\cdots,v_n$, the sequence $(d(v_1),d(v_2),\cdots,d(v_n))$ is called a **degree sequence** of G. A vertex with degree zero is called an **isolated vertex**; a vertex with degree one is a **pendant vertex**. The unique edge incident with a pendant vertex is a **pendant edge**.

Two vertices u and v of G are said to be **connected** if there is a (u, v)–path in G. Thus there is a partition of V into nonempty subsets V_1, V_2, …, V_w such that two vertices u and v are connected if and only if both u and v belong to the same set V_i. The subgraphs $G[V_1],G[V_2],\cdots,G[V_w]$ are called the **components** of G. If G has exactly one component, G is **connected**; otherwise G is **disconnected**. We denote the number of components of G by $\omega(G)$. Two graphs G and H are said to be **isomorphic** (written $G\cong H$) if there are bijections $\theta:V(G)\to V(H)$ and $\phi:E(G)\to E(H)$ such that $\psi_G(e)=uv$ if and only if $\psi_H(\phi(e))=\theta(u)\theta(v)$; such a pair (θ,ϕ) of mappings is called an isomorphism between G and H.

For example, the following graph G and H are isomorphic.

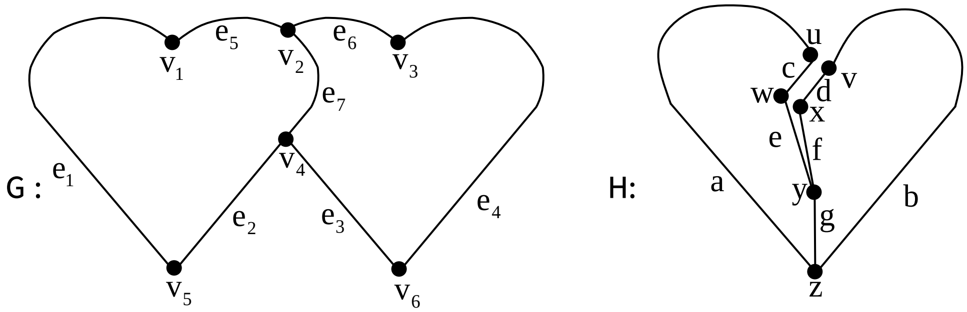


Figure (3) Graphs G and H

So, $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$,

$V(H) = \{u, v, w, x, y, z\}$, $E(H) = \{a, b, c, d, e, f, g\}$.

Let $\theta : V(G) \to V(H)$ and $\phi : E(G) \to E(H)$ be

$\theta(v_1) = w$, $\theta(v_2) = y$, $\theta(v_3) = x$, $\theta(v_4) = z$, $\theta(v_5) = u$, $\theta(v_6) = v$,

$\phi(e_1) = c$, $\phi(e_2) = a$, $\phi(e_3) = b$, $\phi(e_4) = d$, $\phi(e_5) = e$, $\phi(e_6) = f$,

$\phi(e_7) = g$.

Hence, $\psi_G(e_1) = v_1 v_5 \Leftrightarrow \psi_H(\phi(e_1)) = \psi_H(c) = wu = \theta(v_1)\,\theta(v_5)$.

$\psi_G(e_2) = v_4 v_5 \Leftrightarrow \psi_H(\phi(e_2)) = \psi_H(a) = zu = \theta(v_4)\,\theta(v_5)$.

$\psi_G(e_3) = v_4 v_6 \Leftrightarrow \psi_H(\phi(e_3)) = \psi_H(b) = zv = \theta(v_4)\,\theta(v_6)$.

$\psi_G(e_4) = v_3 v_6 \Leftrightarrow \psi_H(\phi(e_3)) = \psi_H(d) = xv = \theta(v_3)\,\theta(v_6)$.

$\psi_G(e_5) = v_1 v_2 \Leftrightarrow \psi_H(\phi(e_5)) = \psi_H(e) = wy = \theta(v_1)\,\theta(v_2)$.

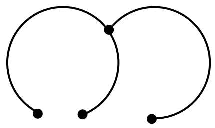
$\psi_G(e_6) = v_2 v_3 \Leftrightarrow \psi_H(\phi(e_6)) = \psi_H(f) = yx = \theta(v_2)\,\theta(v_3)$.

$\psi_G(e_7) = v_2 v_4 \Leftrightarrow \psi_H(\phi(e_7)) = \psi_H(g) = yz = \theta(v_2)\,\theta(v_4)$.

Therefore Graphs G and H are isomorphic, that is, $G \cong H$.

3. Graph of Myanmar Alphabet

Myanmar alphabets are based on the circular curve shape. In this paper, we use the method of drawing Myanmar consonant. (Directions of curves are not considered.) We set as "vertex" to beginning point and end point of curve. And, if a curve is incident to other curve, we take this place as "vertex". The following graphs are graph of Myanmar alphabet shape.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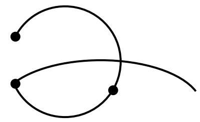
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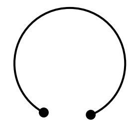
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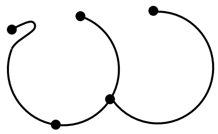
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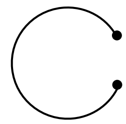
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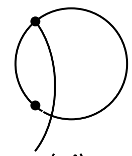
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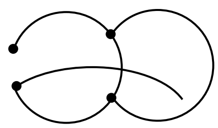
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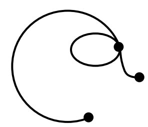
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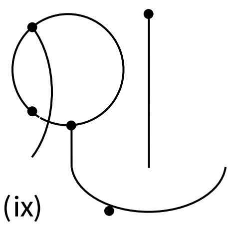
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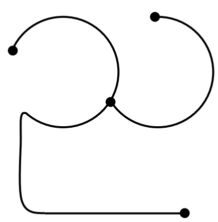
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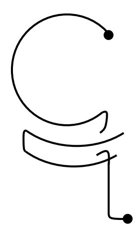
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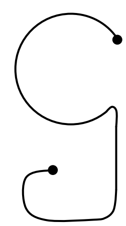
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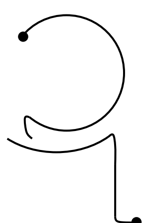
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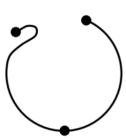
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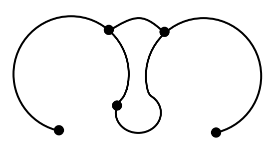
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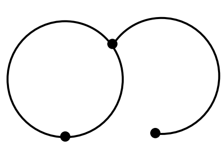
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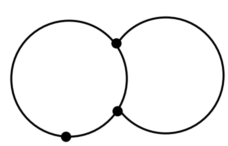
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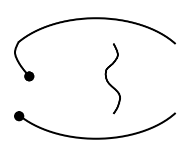
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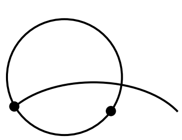
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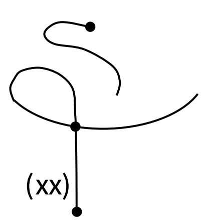
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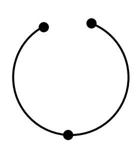
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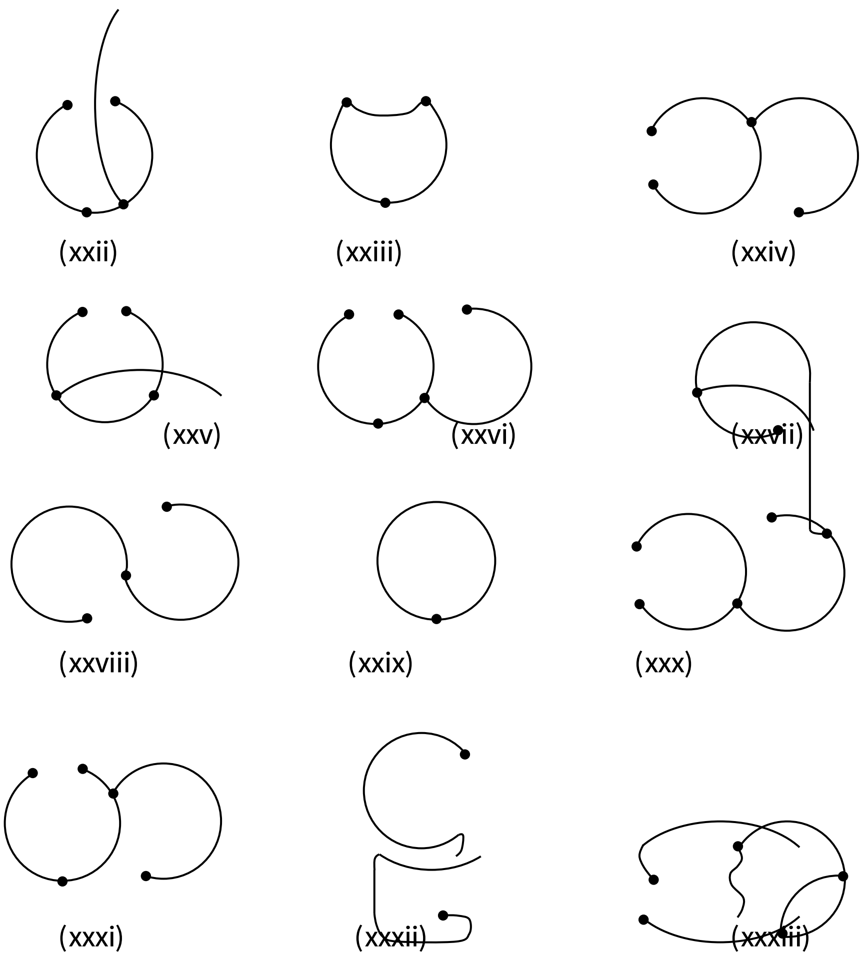

Figure (4) Graphs of letters (Myanmar Alphabet)

From Figure 4 (i) to Figure 4 (xxxiii), all graphs are connected.

4. Labeled Graphs and Isomorphism

In this section, firstly, we denote the vertices which has degree one as $v_1, v_2, \ldots, v_i$, and then the vertices which has degree two as $v_{i+1}, \ldots, v_j$, the vertices which has degree three as $v_{j+1}, \ldots$, respectively, and so on; $1 < i < j < \cdots$. And, we denote the edge which incident with v_1 and v_r $(r \geq 2)$ as e_1. If the edge which incident with v_2 and v_s $(s \geq 3)$, we denote it e_2, and so on.

Example 1

We label the vertices and edges to Figure 4(i) and Figure 4(x).

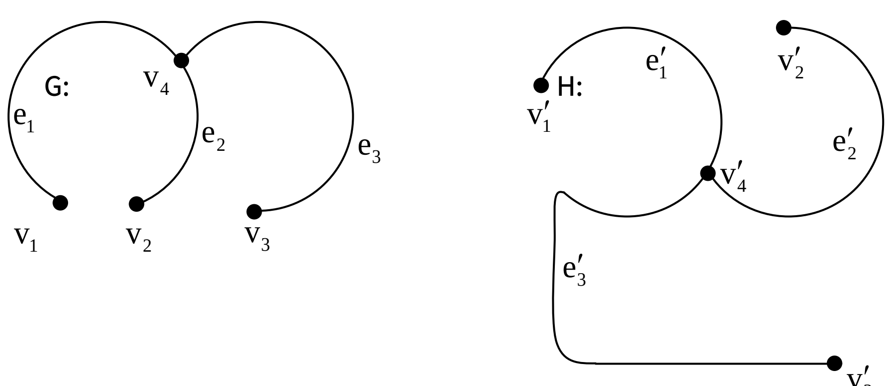

Figure (5) Graphs G and H

So, $V(G) = \{v_1, v_2, v_3, v_4\}$, $E(G) = \{e_1, e_2, e_3\}$,

$V(H) = \{v'_1, v'_2, v'_3, v'_4\}$, $E(H) = \{e'_1, e'_2, e'_3\}$.

Define $\theta : v_i \mapsto v'_i$ and $\phi : e_i \mapsto e'_i$, then

$$\theta(v_1) = v'_1, \quad \theta(v_2) = v'_2 \quad \theta(v_3) = v'_3, \quad \theta(v_4) = v'_4,$$

$$\phi(e_1) = e'_1, \quad \phi(e_2) = e'_2, \quad \phi(e_3) = e'_3.$$

$$\psi_G(e_1) = v_1v_4 \Leftrightarrow \psi_H(\phi(e_1)) = \psi_H(e'_1) = v'_1v'_4 = \theta(v_1)\,\theta(v_4).$$

$$\psi_G(e_2) = v_2v_4 \Leftrightarrow \psi_H(\phi(e_2)) = \psi_H(e'_2) = v'_2v'_4 = \theta(v'_2)\,\theta(v'_4).$$

$$\psi_G(e_3) = v_3v_4 \Leftrightarrow \psi_H(\phi(e_3)) = \psi_H(e'_3) = v'_3v'_4 = \theta(v_3)\,\theta(v_4).$$

Therefore $G \cong H$.

Example 2

We label the vertices and edges to Figure 4(vi) and Figure 4(xix).

Figure (6) Graphs G and H

So, $V(G) = \{v_1, v_2\}$, $E(G) = \{e_1, e_2, e_3\}$,

$V(H) = \{v'_1, v'_2\}$, $E(H) = \{e'_1, e'_2, e'_3\}$.

Define $\theta : V(G) \to V(H)$ by $\theta(v_i) = v'_i$

and $\phi : E(G) \to E(H)$ by $\phi(e_i) = e'_i$.

$$\theta(v_1) = v'_1 \quad \theta(v_2) = v'_2,$$

$$\phi(e_1) = e'_1, \quad \phi(e_2) = e'_2, \quad \phi(e_3) = e'_3.$$

$$\psi_G(e_1) = v_1v_2 \Leftrightarrow \psi_H(\phi(e_1)) = \psi_H(e'_1) = v'_1v'_2 = \theta(v_1)\,\theta(v_2).$$

$$\psi_G(e_2) = v_1v_2 \Leftrightarrow \psi_H(\phi(e_2)) = \psi_H(e'_2) = v'_1v'_2 = \theta(v_1)\,\theta(v_2).$$

$$\psi_G(e_3) = v_1v_2 \Leftrightarrow \psi_H(\phi(e_3)) = \psi_H(e'_3) = v'_1v'_2 = \theta(v_1)\,\theta(v_2).$$

Therefore $G \cong H$.

In the above examples, we can show that there are isomorphic graphs of the letters of Myanmar alphabet. The following examples are some results of isomorphic graphs.

Example 3

If graphs G and H are isomorphic, then n(G) = n(H) and m(G) = m(H).

Indeed,

let G and H are isomorphic. Then there are bijections $\theta : V(G) \rightarrow V(H)$ and $\phi : E(G) \rightarrow E(H)$ such that $\psi_G(e) = uv$ if and only if $\psi_H(\phi(e)) = \theta(u)\theta(v)$. Thus the numbers of elements in domain and codomain are equal as well as the number of edges in G and H are equal, and the number of vertices in G and H are equal. Therefore n(G) = n(H) and m(G) = m(H).

The following example is shown that the converse of Example 3 is false.

Example 4

Graphs G and H are not isomorphic although n(G) = n(H) and m(G) = m(H).

Consider Figure 4(xxii) and Figure 4(xxv).

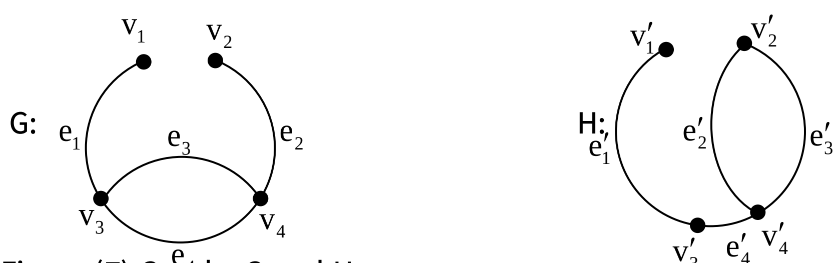

Figure (7) Graphs G and H

So, $V(G) = \{v_1, v_2, v_3, v_4\}$, $E(G) = \{e_1, e_2, e_3, e_4\}$,

$V(H) = \{v_1', v_2', v_3', v_4'\}$, $E(H) = \{e_1', e_2', e_3', e_4'\}$.

Let the bijection $\theta : V(G) \rightarrow V(H)$ is defined by

$$\theta(v_1) = v_1', \quad \theta(v_2) = v_2' \quad \theta(v_3) = v_3', \quad \theta(v_4) = v_4'.$$

Let the bijection $\phi : E(G) \rightarrow E(H)$ is defined by

$$\phi(e_1) = e_1', \quad \phi(e_2) = e_2', \quad \phi(e_3) = e_3', \quad \phi(e_4) = e_4'.$$

We have $\psi_G(e_3) = v_3v_4$, then

$$\psi_H(\phi(e_3)) = \psi_H(e_3') = v_2'v_4' = \theta(v_2)\,\theta(v_4).$$

That is, $\psi_G(e_3) = v_3v_4 \not\Rightarrow \psi_H(\phi(e_3)) = \psi_H(e_3') = v_2'v_4' = \theta(v_2)\,\theta(v_4)$.

Therefore $G \not\cong H$.

Example 5

If graphs G and H have the same degree sequence, then G and H are isomorphic.

In Example 1, the degree sequence of G is (1, 2, 3), and the degree sequence of H is (1, 2, 3). Therefore G and H are isomorphic. In Example 2, the degree sequence of G is (1, 1, 1, 3), and the degree sequence of H is (1, 1, 1, 3). Therefore G and H are isomorphic. In Example 4, the degree sequence of G is (1, 1, 3, 3), and the degree sequence of H is (1, 2, 2, 3). Therefore G and H are not isomorphic.

5. Conclusion

By the definition of isomorphism, degree sequences and labeled graph, we may classify the seven groups. The groups of letters of Myanmar Alphabet are listed as following.

Group (1)	က, ည, ဘ, သ
Group (2)	ခ, ရ
Group (3)	ဂ, င, ဋ, ဌ, ဍ, ဒ, ဎ
Group (4)	ဃ, ယ, ဟ
Group (5)	စ, ဓ
Group (6)	ဇ, န
Group (7)	ဥ, ပ, လ

The rest letters (ဆ, ဈ, ဏ, တ, ထ, ဖ, ဗ မ, ဝ and အ) are not isomorphic to other letters. A graph that is connected together, where all the edges are directed from one vertex to another is called directed graph or digraph. We can further study isomorphic digraph of alphabet, and also classify.

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